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ON THE MIXED PARTIAL DIFFERENTIAL EQUATIONS IN n INDEPENDENT VARIABLES.

par GU Chaohao

The boundary value problems for mixed partial differential equations were thought to be very difficult. However, now we are able to point out a large class of well-posed boundary value problems [1] . Instead of talking about the general methods I prefer to presenting here some special cases [2], [3], [4] .

Let

$$L\phi = (\delta^{ij} - x^i x^j)\,\partial_{ij}\phi + 2ax^i\partial_i\phi - a(a+1)\phi = f \qquad (i,j = 1,2,\dots,n) \tag{1}$$

and Ω be a boundary domain in $\mathbb{R}^n$. The operator L is elliptic inside the unit sphere and hyperbolic outside the unit sphere. Suppose that the $\partial\Omega$ is sufficiently smooth and the tangent planes of $\partial\Omega$ do not meet the unit sphere. We have

Theorem 1 : If $f \in H_s(\Omega)$ and $a > -\frac{n}{2} + s$ $(s > 1)$, there exists uniquely a solution $\phi \in H_{s+1}(\Omega)$ of (1) such that the boundary conditions

$$\phi\big|_{\partial\Omega} = 0 \quad , \quad \frac{\partial\phi}{\partial n}\Big|_{\partial\Omega} = 0 \tag{2}$$

both be satisfied. In particular, if $s \geqslant [\frac{n}{2}] + 2$, the boundary value problem (1), (2) admits a classical solution.

Theorem 2 : If $f \in H_s(\Omega)$ and $a < -\frac{n}{2}$ $(s > 1)$, there exists a solution $\phi \in H_{s+1}(\Omega)$ of (1) uniquely.

Theorem 3 : If (a+1) is not a non-negative integer, then the map $L : \phi \to L\phi$ is an homeomorphism of $C^\infty(\bar{\Omega})$ to $C^\infty(\bar{\Omega})$. If (a+1) is a non-negative integer, then the image of L is a subspace of codimension Na and the dimension of $L^{-1}(0)$ is Na, where $Na = C^{n+a}_{a+1} + C^{n+a-1}_{a}$ (if $a \neq -1$) or 1 (if $a = -1$). Moreover, for an analytic f, $L^{-1}(f)$, if exists, is analytic too.

These theorems indicate that the properties of mixed equations essentially depend on the coefficients of lower order terms and the boundary conditions depend on the functional spaces of the unknown functions.

The similar results were obtained for the operator [3], [4]

$$L(a,b) = (\delta^{ij} - x^i x^j)\partial_{ij}\phi + 2ax^i\partial_i\phi - b\phi \; . \tag{3}$$

Moreover, for the quasilinear equation

$$((1 + \frac{1}{a}\lambda(x,\phi,\partial\phi))\,\delta^{ij} - x^i x^j)\,\partial_{ij}\phi + 2ax^i\partial_i\phi - a(a+1)\phi = f(x,\phi,\partial\phi) \qquad (4)$$

with $\lambda(x,0,0) = 0$ we have

Theorem 4 : If a is sufficiently large the boundary value problem (2), (4) admits a classical solution ϕ .

We note that the equation (1) is hyperbolic near the boundary $\partial\Omega$ and elliptic near the origin. Hence theorem 4 is an existence theorem for an essentially quasilinear mixed equation whose elliptic region and hyperbolic region cannot be determined before the solution is obtained .

For the proof we use the theory of positive symmetric systems and some of its developments [7], [5] .

In [8] some existence theorems of H_1 strong solutions were obtained.

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