Winter Braids Lecture Notes

Paolo Bellingeri, Vincent Florens, Jean-Baptiste Meilhan and Emmanuel Wagner

Introduction

Vol. 2 (2015), p. i-vi.

<http://wbln.cedram.org/item?id=WBLN_2015__2__r1_0>

Winter Braids Lecture Notes
Vol. **2** (2015) i-vi

Introduction

PAOLO BELLINGERI, VINCENT FLORENS, JEAN-BAPTISTE MEILHAN AND EMMANUEL WAGNER

Winter Braids is an international school on low dimensional topology, with a particular emphasis on the multiple aspects of braids: algebraic, geometrical, topological or algorithmic. More generally, it is devoted to the various thematics connected with these objects, such as mapping class groups, singularity theory and plane algebraic curves, quantum topology and TQFTs, symplectic topology, categorification or theoretical physics. The school is held each winter since 2010, in a different french university.

The *Winter Braids Lecture Notes* is a 'gratis' open access electronic and peer-reviewed journal, which contains the lecture notes of each school, since the fourthe edition. The main aim of this journal is to provide a body of original introductory texts for young researchers on classical and emerging topics in low dimensional topology and its ramifications.

This volume of *Winter Braids Lecture Notes* contains lecture notes for the four mini-courses given at Winter Braids V, which took place in Pau from February 16th to 19th, 2015. They were dedicated to TQFTs, quantum representations of mapping class groups, knot theoretical invariants of 3-dimensional vector fields, generalizations of the Alexander polynomial and Artin Tits groups. Below is a short description for each of these courses.

- The mini-course of François Costantino (Toulouse) was conceived as an introduction to Topological Quantum Field Theories (TQFTs) and their motivations. It focused in particular on the universal construction and applications to the study of mapping class groups.

- Pierre Dehornoy (Grenoble) gave three lectures on a particular invariant of vector fields under diffeomorphism, called helicity. The main aim of this course was to give a survey on this invariant, presenting in particular its relations with other (sometimes proportional) known invariants and the state of the art of the research in this field.

- The aim of Juan González-Meneses (Sevilla) was to present some geometric methods in the study of Artin Tits groups. The course was organised as a self contained introduction to braid groups, their combinatorial generalisations (Artin Tits groups) and their actions on geometric objects such as hyperbolic surfaces and complexes of curves.

- Teruaki Kitano (Tokyo) presented an introduction to twisted Alexander polynomials of knots, giving their definition in terms of Fox differential calculus, showing computations for some concrete examples and discussing fundamental properties. The last part of the course was devoted to applications of twisted Alexander polynomials on defining partial orders on knots.

The organisers also selected 11 short talks, providing a wide spectrum of topics.
Several talks were mainly concerned with braid groups, related algebras, generalisations

and applications in low dimensional topology. In particular the talks covered notions such as closures of braids in S^3 and Lens spaces, Hecke algebras, virtual braids and application to polymers and fixed point theory. Other short talks addressed topics such as classical and quantum invariants of knots and 3-manifolds, TQFT, mapping class groups (coverings and quantum representations), categorification (Heegaard Floer and Khovanov homologies) and concordance invariants.
The abstracts of these short talks can be found at the end of this introduction.

We are deeply indepted to the four authors of these lecture notes, for their remarkable work and for helping us to pursue the adventure of editing Winter Braids Lecture Notes.
We also wish to ackowledge the support of CNRS, which provided Winter Braids V with the status of *École Thématique*. We also thank the GDR Tresses, for its constant support since the first edition. This edition was also supported by GDR singularités and ANR projects ModGroup, SUSI and VasKho, as well as Laboratoire de Mathématiques Nicolas Oresme, Institut Fourier, Institut Mathématique de Bourgogne and Université de Pau.

Université de Caen, LMNO, UMR 6139, 14032 Caen, France • paolo.bellingeri@unicaen.fr

Université de Pau et des Pays de l'Adour, LMAP, UMR 5142, 64013 Pau, France • vincent.florens@univ-pau.fr

Université Grenoble Alpes, IF, UMR 5582, 38000 Grenoble, France • jean-baptiste.meilhan@ujf-grenoble.fr

Université de Bourgogne, IMB, UMR 5584, 21078 Dijon, France • emmanuel.wagner@u-bourgogne.fr

Participants

Aguilera	Marta	(Universidad de Sevilla)
Albertas	Dvirnas	(Kaunas University of Technology)
Alfieri	Antonio	(Università di Pisa)
Anghel	Cristina Ana-Maria	(Université Paris 7)
Barthel	Senja	(Imperial College)
Bellingeri	Paolo	(Université de Caen Basse-Normandie)
Bénard	Léo	(Université Paris 6)
Ben Aribi	Fathi	(Université Paris 7)
Berrin	Senturk	(Bilkent University)
Casteluber Laass	Vinicius	(Universidade de Sao Paulo)
Chterental	Oleg	(University of Toronto)
Cisneros de la Cruz	Bruno	(Université de Bourgogne)
Conway	Anthony	(Université de Genève)
Corbineau	Kévin	(Université de Grenoble)
Collari	Carlo	(Università di Firenze)
Costantino	Francois	(Université Toulouse 3)
Damiani	Celeste	(Université de Caen Basse-Normandie)
Dehornoy	Pierre	(Université de Grenoble)
De Renzi	Marco	(Université Paris 7)
Detcherry	Renaud	(Université Paris 6)
Ffitch	Edward	(University of Warwick)
Florens	Vincent	(Université de Pau et des Pays de l'Adour)
Gainullin	Fyodor	(Imperial College)
Gale	Nicholas	(University of Warwick)
Geneste	Olivier	(Université de Bourgogne)
Gobet	Thomas	(TU Kaiserslautern)
González-Meneses	Juan	(Universidad de Sevilla)
Guerville-Ballé	Benoît	(Université de Grenoble)
Gunes	Cisem	(Bilkent University)
Kitano	Teruaki	(Soka University)
Kohli	Ben-Michael	(Université de Bourgogne)
Korinman	Julien	(Université de Grenoble)
Liechti	Livio	(Universität Bern)
Nasybullov	Timur	(Novosibirsk State University)
Manfredi	Enrico	(Università di Bologna)
Martel	Jules	(Université Paris 6)
Marzioni	Simone	(Aarhus University)
Magot	Jean-Mathieu	(Université de Grenoble)
Meilhan	Jean-Baptiste	(Université de Grenoble)
Misev	Filip	(Universität Bern)
Morvan	Xavier	(Université de Genève)
Moussard	Delphine	(Università di Pisa)
Palafox	Jordi	(Universié de Pau et des Pays de l'Adour)
Robert	Louis-Hadrien	(Universität Hamburg)
Santharoubane	Ramanujan	(Université Paris 7)
Serrano de Rodrigo	Juan	(Université de Pau et des Pays de l'Adour)
Silvero	Marithania	(Universidad de Sevilla)
Thillaisundaram	Anitha	(Düsseldorf Universität)
Tokunaga	Hiroo	(Tokyo Metropolitan University)
Valladares	Loles	(Universidad de Sevilla)

Viu Sos	Juan	(Université de Pau et des Pays de l'Adour)
Wagner	Emmanuel	(Université de Bourgogne)
Wedrich	Paul	(University of Cambridge)
Wenzel	Ansgar	(University of Sussex)

Abstracts of Short Talks

Marta Aguilera (Univ. Sevilla)
Train tracks of rigid braids

The braid group acts on the curve complex of the n-times punctured disc. If a braid is pseudo-Anosov, its action has a north-south dynamic where the attracting limit point corresponds with the stable foliation. Bestvina and Hendel gave an algorithm to compute a graph, called train track, which encodes the foliation and the action of the braid. Their algorithm works for mapping class groups in general. We will show how one can compute a train track in a different way, in the particular case of (so- called) rigid braids. As every pseudo-Anosov braid has a conjugate one of whose powers is rigid, this allows to compute a train track related to any pseudo-Anosov braid. Our aim is to explain how, using the Garside structure of braid groups, one can show that rigid braids admit particularly simple train tracks.

Senja Barthel (Imperial College)
Braids in Crystals

Coordination polymers (think of crystals) can have different topological modes. The modes of 1-dimensional coordination polymers have a description as braids. We predict all possible entanglements in those molecules. This is joint work with Davide M. Proserpio, F. Din-Houn Lau and Igor Baburin.

Vinicius Casteluber Laass (Sao Paolo)
The Borsuk-Ulam problem for homotopy class of functions - An approach using braid groups

The well-known Borsuk-Ulam theorem states that given any continuous map $f : S^2 \to \mathbb{R}^2$, there is a point x in S^2 such that $f(-x) = f(x)$. One possible generalization of this theorem is to consider other spaces and involutions. If $\tau : M \to M$ is a free involution, we say that a triple (M, τ, N) has the Borsuk-Ulam property if for every continuous function $f : M \to N$, there is a point $x \in M$ such that $f(\tau(x)) = f(x)$. So if (M, τ, N) does not have the Borsuk-Ulam property, there exists a continuous map $f : M \to N$ such that $f(\tau(x)) \neq f(x)$ for every x in M. A natural question that arises is the following: up to homotopy, how many functions are there with this property ? In this talk, I will show a relation between this problem and a diagram involving braid groups, in the case that M and N are compact surfaces without boundary, and I will give some examples.

Oleg Chterental (Univ. Toronto)
Virtual braids and virtual curve diagrams

We define a faithful action of the virtual braid group VB_n on certain planar diagrams called virtual curve diagrams. Our action is similar in spirit to the Artin action of the braid group B_n on the free group F_n and it provides an easy combinatorial solution to the word problem in VB_n.

Fyodor Gainullin (Imperial College)
Heegaard Floer homology, Alexander polynomial and Dehn surgery on alternating knots

One of the biggest challenges in modern low-dimensional topology is to understand what knots can yield a given 3-manifold by Dehn surgery. There do exist 3-manifolds, which can be obtained by Dehn surgery on infinitely many distinct knots. However, it turns out, that for every 3-manifold there are only finitely many alternating knots that can give it by surgery. I will attempt to go through the main steps of the proof, giving a rudimentary description of relevant tools from Heegaard Floer homology.

Thomas Gobet (TU Kaiserslautern)
Dual braid monoids and Hecke algebras

An understanding of the Hecke algebras using dual braid monoids would be of interest, especially for the complex reflection groups, since there is no canonical positive braid monoid. On the other hand, still in the Coxeter case, nothing has been done in this direction. We will review what is already known in the topic and give new recent results.

Enrico Manfredi (Univ. Bologna)
Links in lens spaces with inequivalent lift

A strong geometric invariant of links in lens spaces is the lift, that is to say a link in the 3-sphere that is the pre-image of a link under the universal cover of the lens space; the lift is clearly a freely periodic link. An interesting question is whether the lift is a complete invariant of links in lens spaces. Several counterexamples to this question are produced using braids. A class of links in lens spaces can be easily described by a braid, and so their lift. A short tabulation of the possible lifts gives two pairs of links with equivalent lift. Moreover cabling one of these examples with palindromic braids produces an infinite family with such a property.

Filip Misev (Univ. Bern)
Cutting and gluing fibre surfaces

A classical theorem in the theory of fibred links states that the fibering Seifert surfaces of any two such links in the three-sphere are related by a sequence of so-called Hopf plumbings and deplumbings. The aim of the talk is to explore this relation and to give examples that illustrate how (non-)unique such plumbing sequences can be.

Ramanujan Santharoubane (Univ. Paris 7)
On the AMU conjecture for the four holed sphere

We are going to focus on quantum representations of the mapping class group of the four holed sphere arising from the Witten-Reshetikhin-Turaev $SU(2)$ Topological Quantum Field Theory (TQFT). We will see how to extend the result concerning the asymptotic behavior of pseudo-Anosov elements proved by J.E Andersen, G. Masbaum and K. Ueno.

Marithania Silvero (Univ. Sevilla)
Positivity of Conway polynomials of closed BKL-positive 3-braids

In 1989, Peter Cromwell proved that positive links have positive Conway polynomial; that is, all the coefficients are non-negative. Positive links include those links which are closure of positive braids in terms of Artin generators. A link is said to be BKL-positive if it can be expressed by a positive braid word using the generators introduced by Birman, Ko and Lee in 1998. Not every BKL-positive link is positive. In this talk we show that BKL-positive links with braid index 3 have positive Conway polynomial.

Paul Wedrich (Univ. Cambridge)
Deformations of link homologies

I will start by explaining how deformations help to answer two important questions about the family of (colored) $sl(N)$ link homology theories: What geometric information about links do they contain? What relations exist between them? I will recall results of Lee, Gornik and Wu on generic deformations of $sl(N)$ link homologies and sketch how they generalize to include the case of non-generic deformations. The result is a decomposition theorem for deformed colored $sl(N)$ link homologies which leads to new spectral sequences between various type A link homologies and concordance invariants in the spirit of Rasmussen's s-invariant. Joint work with David Rose.