Winter Braids Lecture Notes

Participants & Programme

Vol. 9 (2023), p. 1-6.

https://doi.org/10.5802/wbln.41

e-ISSN : 2426-0312

Winter Braids Lecture Notes
Vol. **9** (2023) 1-6

Participants & Programme

This volume of *Winter Braids Lecture Notes* contains the lecture notes for the four mini-courses given at Winter Braids XII, which took place at the university of Tours, from February 21th to 24th, 2023. We are grateful to Thomas Gobet, for his great job as local organizer for this edition of the Winter Braids school.

Participants

Bakshi	Rhea Palak	(ETH Zürich)
Becerra	Jorge	(University of Groningen)
Bellingeri	Paolo	(Université Caen Normandie)
Benard	Léo	(University of Göttingen)
Bosch	Boudewjin	(University of Groningen)
Brisson	Paul	(Université de Nantes)
Brown	Jennifer	(Yale University)
Capovilla Searle	Michele	(University of Iowa)
Cardenas Andaur	O’bryan	(Universidad De Sevilla)
Chatterjee	Rima	(University Of Cologne)
Chemin	Hugo	(Université Caen Normandie)
Colin	Nestor	(Cinvestav)
Collari	Carlo	(Università di Pisa)
Damiani	Celeste	(Istituto Italiano di Tecnologia)
Darné	Jacques	(UC Louvain)
Del Valle	Alvaro	(Universidad de Sevilla)
De Mesmay	Arnaud	(Université Gustave Eiffel)
Di Petrillo	Gemma	(Università di Trento)
Di Prisa	Alessio	(Università di Pisa)
Dlugie	Ethan	(UC Berkeley)
Faes	Quentin	(Tokyo University)
Fanoni	Federica	(Paris Est)
Feingesicht	Edouard	(Université Caen Normandie)
Feller	Peter	(ETH Zurich)
Ferretti	Livio	(Universität Bern)
Forget	Hugo	(Université de Bourgogne)
Florens	Vincent	(Université de Pau)
Framba	Giovanni	(Università di Pisa)
Garnier	Owen	(Université d’Amiens)
Gavazzi	Federica	(Université de Bourgogne)
Gobet	Thomas	(Université de Tours)
Graff	Emmanuel	(Université Caen Normandie)
Guoyi Chen	Jacopo	(SNS Pisa)

Gurnari	Davide	(IMPAN)
Haioun	Benjamin	(Université Toulouse)
Haladjia	Igor	(Université de Tours)
Heng	Edmund	(IHES)
Ibarra	Dione	(Monash University)
Kegel	Marc	(Humboldt University)
Kohn	Michael	(Durham University)
Lapie	Quentin	(ENS Lyon)
Lecouvey	Cedric	(Université de Tours)
Lou	Tianyi	(Université Côte d'Azur)
Makri	Stavroula	(Nicolaus Copernicus University)
Merz	Alice	(Università di Pisa)
Mandin-Hublé	Yohann	(Université Grenoble Alpes)
Manouras	Manousos	(Université de Pau)
Mazet	Laurent	(Université de Tours)
Montoya Vega	Gabriel	(City University of New York)
Nanda	Neha	(Université Caen Normandie)
Paolini	Giovanni	(Caltech and Amazon)
Rizzi	Edoardo	(SNS Pisa)
Rodau	Adrien	(Université de Pau)
Santoro	Diego	(SNS Pisa)
Schelstraete	Leo	(UC Louvain)
Sienicka	Agata	(University of Bonn)
Silvero	Marithania	(Universidad de Sevilla)
Soergel	Mireille	(Université De Bourgogne)
Stenhede	Eric	(University of Vienna)
Truol	Paula	(ETH Zurich)
Van Helden	Kevin	(University of Groningen)
Vera	Anderson	(Institute of Basic Science CGP)
Verchinine	Vladimir	(Université de Montpellier)
Wagner	Emmanuel	(IMJ PRG)
Wang	Bangxin	(University of Zurich)
Zhang	Yibo	(Université Grenoble Alpes)

Abstracts of Courses

Federica Fanoni (Paris Est)
Big mapping class groups

Big mapping class groups are mapping class groups of surfaces of infinite type (such as surfaces of infinite genus). After introducing these objects, I will discuss some algebraic properties of big mapping class groups. I will then recall the Nielsen-Thurston classification of mapping classes for finite-type surfaces and discuss the problem of generalizing this classification to the infinite-type setting.

Peter Feller (ETH Zurich)
Positive braids and concordance

Braids can be understood from different points of view all leading to different subfields of geometric topology. The standard group presentation leads to the study of Artin groups, geometric braids relate to knot theory, and braid groups are mapping class groups.

All these points of view are featured in courses at this conference, but we will focus on geometric braids and their relation to knot theory and the study of manifolds of dimension three and four. Rather than providing a general overview, in this course we use positive braids as inspiration for a tasting menu in low-dimensional topology. Whatever your interest in braids, in this course I hope to show you that their topology and combinatorics emerge naturally.

Concrete topics that we will discuss:

- An introduction to braids and the above mentioned points of views
- Geometric braids as a tool to study knots and links via the closure operation
- Links of plane curve singularities as closures of positive braids
- The notions of slice knots, concordance, and the raison d'être for low-dimensional topology (in contrast to high-dimensional topology)
- Thom Conjecture, local Thom Conjecture, and the slice-Bennequin inequality.
- Applications (as time permits): Exotic $\mathbb{R}^4$'s, Abhyankar-Moh-Suzuki theorem and a description of the algebraic automorphisms of $\mathbb{C}^2$.

This course does not assume any knowledge of the topics mentioned above.

Giovanni Paolini (Caltech and Amazon)
The $K(\Pi, 1)$ conjecture for Artin groups

In this mini-course, I will introduce the $K(\Pi, 1)$ conjecture for Artin groups and give an overview of its solution in the spherical and affine cases, using Garside structures and other techniques of combinatorial topology.

Special Lecture

Arnaud De Mesmay (Université Gustave Eiffel)
Algorithms and complexity in low-dimensional topology
The aim of this lecture is to provide an introduction to some algorithmic and complexity questions in low-dimensional topology. We will start by explaining the theory of normal surfaces and how it can be used to provide both theoretical and practical algorithms for many problems surrounding knots and 3-manifolds, most famously unknot and 3-sphere recognition. Analyzing these algorithms will naturally lead us to introducing basic complexity classes and hardness reductions, and wondering where various low-dimensional topology problems fit when viewed through the lens of computational complexity. While there has been a flurry of results in this theme in the past years, there are still tremendous gaps between most known upper and lower bounds, and we will emphasize the numerous open problems in the area.

Abstracts of Short Talks

Léo Bénard (University of Göttingen)
Finite type invariants and random walks on triangulations

We exhibit some topological invariants as the order of a pole of series – hence out of their convergence domain – whose coefficients are expressed in terms of random walks on a triangulation of a given manifold. Joint work with Yann Chaubet and Viet Dang.

Rima Chatterjee (University Of Cologne)
On a structure theorem of Legendrian knots

Structure theorems of a Legendrian knot have been an interesting study in contact geometry. One can ask when topological operations on a knot gives us important information about the geometry or under what conditions the geometric features of the knot are preserved under certain topological constructions. In this talk I'll explore this question and focus on a recent work on answering when does cabling operation preserves the special geometric property of a knot in an overtwisted contact manifold. This is a joint work with Etnyre, Min, and Mukherjee.

Carlo Collari (Università di Pisa)
Some obstruction to positivity conditions for knots and links

This talk concerns the problem of obstructing concordance and equivalence to variously positive links. The talk will be organised as follows; first, we give an overview on positivity conditions for links and present the problem. Then, we describe some obstructions based on link polynomials, slice-torus link invariants and combinatorial invariants. Finally, we compare these results and show some examples.

Celeste Damiani (Istituto Italiano di Tecnologia)
A topological mammography clock

The amount of fibroglandular tissue within the breast is one of the strongest risk factors for breast cancer. It is often measured using breast x-rays (mammograms) done during routine breast cancer screening appointment. Mammographic density is often measured visually by trained readers as the proportion of radio-opaque tissue on the mammogram.

Breast density decreases with age so it is possible to predict a woman's age from her breast density. We aim to improve risk prediction of future breast cancer by using this to estimate 'breast age' from the mammogram, and compare it to a woman's actual age at the time of the mammogram. We hypothesize that women whose breast age is younger than their actual age will be at increased risk of breast cancer.

The issue that we consider in this talk is how to measure breast age. We do so harnessing Persistent Homology (PH) based on information from mammography images, that is then fed into a traditional computer vision/machine learning pipeline.

Improved risk assessment may be clinically useful because there are increasing moves to replace one-size-fits-all mammography screening for breast cancer by risk-adapted screening, where both the frequency of screening and the modality are chosen based on risk of breast cancer.

PH is a tool used in topological data analysis (TDA) to study qualitative features of data. It is robust to perturbations of the input data, independent of dimensions and coordinates, and provides a compact representation of the qualitative features of the input. It can be encoded in the form of Persistence Barcodes (PB). In this talk, we use an image-based approach to build several PB representations for each mammogram, based on a landmark selection method known as local binary patterns, that encode different types of local texture and structure from a digital image. We will present some results of a classification experiment where we predict the age of a patient from their mammogram.

This work is part of the CRUK funded project (reference 49757/A28689) “An Artificial Intelligence System for Real-time Risk Assessment at Mammography Screening (Mammo AI)”.

Jacques Darné (UC Louvain)
Lower central series of partitioned braids on surfaces

Partitioned braid groups (sometimes called “mixed braid groups”) are subgroups of the braid group standing between the pure braid group P_n and the whole braid group B_n. On the one hand, the lower central series of B_n is almost trivial. On the other hand, the lower central series of P_n is a very rich object, encoding finite type invariants of braids. As a consequence, one can expect partitioned braid groups to display a range of intermediate behaviors, and this is indeed what we observe. In this talk, we will explore these different behaviours and give an answer to the first question one can ask about these lower central series : when do they stop ? Even this simple question turns out to be a difficult one, especially when one considers its generalization to braids on surfaces. However, we can answer it almost completely, leaving open only some cases of partitioned braids on the projective plane.

Owen Garnier (Université d’Amiens)
Bessis category of the complex braid group B31

In his seminal paper on complex reflection arrangements, David Bessis introduces a general Garside structure for braid groups of well-generated complex reflection groups. The study of this Garside structure has been very fruitful in the proof of many properties of those braid groups (for instance, the determination of the centers of all finite index subgroups). A comparable structure was defined, but not studied, by Bessis for dealing with some badly-generated group, especially B31, which is the only exceptional badly-generated braid group of rank higher than 2. The cost is that this structure is a category, and not a monoid. In this talk we give the definition of the Garside category associated to B31, and we give some details as to how it can be used to compute a presentation of B31, and to determine the centers of its finite index subgroups (these two results were known to hold, but no proof involving a Garside structure were known).

Davide Gurnari (IMPAN)
Exploring relations between knots invariants using Mapper algorithms

Mapper-type algorithms are useful tools for revealing the shape of high dimensional data and visualising functions over them. Moreover, new versions of such algorithms that incorporate the symmetries within the data provide ways to compare various descriptors. In this presentation, I will discuss how this tools, combined with machine learning techniques, can be used to explore the relations between different knot polynomials. This is joint work with Paweł Dłotko and Radmila Sazdanovic.

Emmanuel Graff (Université Caen Normandie)
On braids and links up to link-homotopy

This talk deals with links and braids up to link-homotopy, studied from the viewpoint of Habiro’s clasper calculus. More precisely, we use clasper homotopy calculus in two main directions. First, we define and compute a faithful linear representation of the homotopy braid group, by using claspers as geometric commutators. Second, we give a geometric proof of Levine’s classification of 4-component links up to link-homotopy, and go further with the classification of 5-component links in the algebraically split case.

Edmund Heng (IHES)
Stability conditions and Artin-Tits groups

It is known that certain Artin-Tits groups (Type E) do not possess faithful action on surfaces. On the other hand, these Artin-Tits groups possess faithful actions on triangulated categories. The relevance between these two (seemingly unrelated) facts is that triangulated categories “behave” like surfaces. This is built upon a striking connection between the theory of Bridgeland stability conditions (on triangulated categories) and Teichmuller theory. In this talk, I shall discuss some potential applications of this connection to Artin-Tits groups.

Léo Schelstraete (UC Louvain)
Odd Khovanov homology and higher representation theory

Khovanov homology is an invariant of links generalizing the Jones polynomial in a precise sense: one says that it is a categorification. On the other hand, the Jones polynomial is a quantum invariant, closely related to the representation theory of the quantum group $U_q(\mathfrak{sl}_2)$. These approaches can be unified: there exists a categorification of the representation theory of $U_q(\mathfrak{sl}_2)$, which in turn describes Khovanov homology. Surprisingly, there exists another categorification of the Jones polynomial, called odd Khovanov homology. Its connection to quantum groups is much less understood, in sharp contrast with its even counterpart. In this talk, we will give an overview of the above, pointing toward recent work (j.w.w. Pedro Vaz) giving such a representation theoretic construction of odd Khovanov homology.

Mireille Soergel (Université De Bourgogne)
The geometry of Dyer groups

Dyer groups are a class of groups which have the same solution to the word problem as Coxeter groups and right-angled Artin groups. Both these classes are known to be CAT(0). Indeed Coxeter groups act geometrically on the Davis-Moussong complex and right-angled Artin groups act on the Salvetti complex. After introducing a standard presentation for Dyer groups, we will give a generalization of the Davis-Moussong complex and the Salvetti complex.

Paula Truöl (ETH Zurich)
Strongly quasipositive knots are concordant to infinitely many strongly quasipositive knots

We show that every non-trivial strongly quasipositive knot is smoothly concordant to infinitely many (pairwise non-isotopic) strongly quasipositive knots. In contrast to our result, Baader, Dehornoy and Liechti showed that every concordance class contains at most finitely many positive knots. Moreover, it was conjectured by Baker that smoothly concordant, strongly quasipositive, and fibered knots are isotopic. Our construction uses a satellite operation with companion a slice knot with maximal Thurston-Bennequin number -1.