WBLN

Winter Braids Lecture Notes

Participants & Programme

This volume of *Winter Braids Lecture Notes* contains the lecture notes for the four mini-courses given at Winter Braids IX, which took place at in Reims from March 4th to 7th, 2019. We thank Loïc Poudain d'Andecy for his great work as local organizer for this edition of the Winter Braids school.

Participants

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Yu	Rupert	(Université de Reims)
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Abstracts of Courses

François Dahmani (Université Grenoble Alpes)
Group and subgroups of Interval Exchange Transformations

Given the interval $[0, 1)$, an interval exchange transformation is a bijective piecewise translation from $[0, 1]$ to itself, with finitely many discontinuity points, and by convention, left-continuous. The set of all these transformations form a group, $IET([0, 1))$, which is an interesting test case, and also a case that appears in different situations. We would like to understand more this group. We will illustrate that it is hard to find free subgroups. Actually, to this date, it is not known whether there are any free subgroup in it. We will define a topological "minimal" model for interval exchange transformations, which allows to prove that no subgroup in $IET([0, 1))$ has distorted elements. We will illustrate also the constraints put on certain solvable subgroups of $IET([0, 1))$, in particular that torsion free finitely generated solvable subgroups of $IET([0, 1))$ are virtually abelian. If time permits, in the last lecture, we will make the connection between certain subgroups of $IET([0, 1))$, and certain topological full groups, showing that under certain restrictions on their translation numbers, those subgroups are amenable.

Brendan Owens (Glasgow University)
Knot theory and 4-manifolds

This lecture course will focus on the use of double branched covers in low-dimensional topology, and in particular for concordance and cobordism questions about alternating knots and links. Specific material to be covered will include the Gordon-Litherland pairing on a spanning surface and a generalisation to slice surfaces, the characterisation of alternating links due to Greene and Howie, Liscas classification of slice two-bridge knots, McCoys classification of alternating knots with unknotting number one, and a computer search due to Owens and Swenton for slice alternating knots.

Anne Pichon (AMU, Marseille)
(bi)-Lipschitz geometry of singularities

It is well known that a real or complex analytic germ $(X, 0) \subset (\mathbb{R}^N, 0)$ is topologically conical, i.e., homeomorphic as an embedded variety to the real cone over its link $X^{(\varepsilon)} = X \cap S_\varepsilon^{n-1}$. Now, a natural question is to study the metric evolution of the links $X^{(\varepsilon)}$ when ε converges to zero: how do various regions shrink to zero ? A natural problem is then to build classifications of the germs up to local bi-Lipschitz homeomorphism.

After a general introduction on Lipschitz geometry of singular germs, the course will focus on complex curves and surfaces. I will first present the complete classification of Lipschitz geometry of plane curve germs by showing that the outer Lipschitz of a plane complex curve $(C, 0) \subset (\mathbb{C}^2, 0)$ determines and is determined by the embedded topology its link $C^{(\varepsilon)} = C \cap S_\varepsilon^3$. The core of the proof is based on what we call a 'bubble trick', which enables one to discover the topology of the curve from its Lipschitz geometry, and which is is also used in higher dimensions. Then I will present some results on Lipschitz geometry of complex surfaces. I will show how the Lipschitz geometry of $(X, 0)$ for the inner metric can be described from a specific JSJ (Jaco-Shalen-Johannson) decomposition of $X^{(\varepsilon)}$. The course will be illustrated by many examples.

Hoel Queffelec (IMAG, Montpellier)
Polynomial link invariants and quantum algebras

The definition of the Jones polynomial in the 80's gave rise to a large family of so-called quantum link invariants, based on quantum groups. These quantum invariants are all controlled by a certain

two-variable invariant (the HOMFLY-PT polynomial), which also specializes to the older Alexander polynomial. Building upon quantum Schur-Weyl duality and variants of this phenomenon, I'll explain an algebraic setup that allows for global definitions of these quantum polynomials, and discuss extensions of these quantum objects designed to encompass all of the mentioned invariants.

Abstracts of Short Talks

Leo Benard (Université de Genève)
An extension of the Casson-Lin invariant to links

The Casson invariant is an invariant of integral homology 3-spheres defined as a signed count of irreducible representations of their fundamental group into $SU(2)$. Xiao-Son Lin defined a similar invariant for knot complements, and showed it coincide with the knot signature. In this talk we will extend this construction to links, and prove that this new invariant equals the multivariate Cimasoni-Florens signature for a class of links. This is a work in collaboration with Anthony Conway.

Anthony Conway (Durham University)
Twisted Blanchfield forms, twisted signatures and Casson-Gordon invariants

Given a knot and a representation of its group, the goal of this talk is to describe a new "signature function". In the abelian case, this invariant recovers the Levine-Tristram signature, while in the metabelian case it is closely related to the Casson-Gordon invariants. Applications to knot concordance will then be discussed. This is joint work with Maciej Borodzik and Wojciech Politarczyk.

Jacques Darné (Institut Fourier, Grenoble)
The Andreadakis problem for some subgroups of $Aut(F_n)$

The Andreadakis problem consists in comparing two filtrations on the group IA_n of automorphisms of the free group acting trivially on its abelianization. This difficult problem can be much easier when restricted to some subgroups of IA_n. Especially when these groups decompose nicely as iterated semi-direct products. For example, the pure braid group embeds into IA_n; the Andreadakis filtration restricts to the filtration defined by Milnor invariants there.

Renaud Detcherry (Michigan State University)
Growth rate of Turaev-Viro invariants and volume

The Chen-Yang volume conjecture states that the exponential growth rate of Turaev-Viro invariants of a 3-manifold is its hyperbolic volume. Supporting this conjecture, we show an inequality between volume and the growth rate of Turaev-Viro invariants for arbitrary 3-manifolds, and sharper inequalities for "generic" 3-manifolds.

Honghao Gao (Institut Fourier, Grenoble)
Augmentations and link group representations

Given a framed link, paths in the link complement generate a non-commutative algebra as a link invariant, named the framed cord algebra. Augmentations are rank one representations of this algebra.

We demonstrate how to construct a representation of the link group from an augmentation, and explain the story from symplectic geometry behind the construction.

Filip Misev (Max Planck Institute, Bonn)
Lipschitz normal embedding among superisolated singularities

Locally near a singularity, an algebraic variety X in $\mathbb{C}^n$ is embedded as the cone over a link. This is a classical theorem about the topology of singularities. However, these cones are rarely "Lipschitz normally embedded" that is, metrically conical: the Euclidean distance in the ambient $\mathbb{C}^n$ is usually not comparable (up to Lipschitz constants) to the inner distance given by minimising path length in X. In fact, an irreducible plane curve is Lipschitz normally embedded if and only if it is smooth. I will present an infinite family of superisolated hypersurface singularities in $\mathbb{C}^3$ which are Lipschitz normally embedded. Joint work with Anne Pichon.

Louis-Hadrien Robert (Université de Genève)
A foamy categorification of the Alexander polynomial

The Alexander polynomial, has been categorified using symplectic geometry: this is Heegaard-Floer homology. In this talk I will speak about an another approach to categorify this polynomial. The idea is to see the Alexander polynomial as a member of the big family of quantum invariants. I will show one can use foams to provide construct this new categorification (conjecturally isomorphic to the Heegaard-Floer). Joint work with Emmanuel Wagner.

Marithania Silvero (Polish Academy of Sciences, Warszawa)
Braid combing in polynomial time and space

Braid combing is a procedure defined by Emil Artin to solve the word problem in braid groups. Despite it is conceptually simple, it becomes impractical when computing concrete cases. In fact, it is well-known that (classic) braid combing has exponential complexity. In this talk we will use straight line programs to give a polynomial algorithm which performs braid combing. Moreover, this procedure provides the first algorithm which gives a solution for the word problem in braid groups on surfaces with boundary in polynomial time and space.

Ray Arunima (Max Planck Institute, Bonn)
The 4-dimensional sphere embedding theorem

The disc embedding theorem for simply connected 4-manifolds was proved by Freedman in 1982 and forms the basis for his proofs of the topological h-cobordism theorem, the topological 4-dimensional Poincaré conjecture, 4-dimensional topological surgery, and the classification of simply connected 4-manifolds. The disc embedding theorem for more general manifolds is proved in the book of Freedman and Quinn. However, the geometrically transverse spheres claimed in the outcome of the theorem are not constructed. We close this gap by constructing the desired transverse spheres. We also outline where transverse spheres appear in surgery and the classification of 4-manifolds and give a general 4-dimensional sphere embedding theorem. This is a joint project with Mark Powell and Peter Teichner.

Nora Gabriella Szoke (EPFL Lausanne)
Extensive amenability and its applications

A group action is called amenable if there exists an invariant mean on the space. In this talk I will present a stronger property, namely the extensive amenability of group actions. This property was introduced by Juschenko and Monod, they used it to construct the first examples of finitely generated infinite amenable simple groups. We will see some applications for topological full groups and certain

subgroups of the Interval Exchange Transformations.